LETTERS TO THE EDITOR

To the Editor:

In the article titled "Buoyancy-Driven Circulation in Bubble Columns: Alternative Analysis" (Nov. 1998), Zhang et al. purport to provide an alternative analysis to our and others' works regarding circulation in bubble columns. They first seek to find the inversion position (not the "reversion" point), which demarcates the upflowing central core from the downflowing wall velocity, so that the continuous velocity takes a value of zero at this nondimensional radial position δ (the inversion point).

Our previous works (as cited in Zhang et al.) used the actual velocity profile to find this point δ. Zhang et al., however, "assume," without any proof or empirical evidence, that setting the radial gradient of stress tensor to zero will yield the inversion point. This is manifestly incorrect, and it is no wonder that this erroneous argument leads to values of δ, which differ from ours. In fact, if they should perform the differentiation of the stress tensor (their Eq. 5), they would obtain two terms that also must be identically zero: the derivative of the tensor and the tensor itself must be zero, the former they define as γ (the maximum Reynolds stress point) and the latter as λ (the maximum downflow velocity point). One might suspect then that their "new" definition of δ might be a sort of average of γ and λ, which their Table 1 seems to indicate. All computations that use this erroneous value of δ are also incorrect. Only the velocity profile can yield the value of δ, but, of course, one must obey the rules of continuity.

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Reply:

We appreciate Prof. Rice's interest in our article, but we would like to offer the following explanations to his comments.

(1) Other than the zero radial gradient of the stress T, as pointed out by Prof. Rice, we set the zero radial gradient of the force ξT to specify the inversion point. The latter assumption implies that a static equilibrium (the pressure solely balanced by the buoyancy) is attained at the inversion point, in which the liquid velocity is zero.

(2) In our article, the inversion point δ is predicted by the derived relation (Eq. 12) with known λ and m. It is not an average of λ and γ, as suggested.

(3) Fluid flows can be analyzed from dynamic and/or kinematic points of view; therefore, we believe that there are alternative ways to find a velocity profile or a characteristic point on it. The following example by Burns and Rice (1997) explains the local Reynolds shear stresses as:

$$\tau_C(\xi) = \xi \cdot \left[\frac{\rho_l g R}{2} (p' - 1 + \epsilon_C) \right]$$

for the core region $0 \leq \xi \leq \delta$, and

$$\tau_B(\xi) = \left(\frac{\lambda^2}{\xi} - \xi \right) \cdot \left[\frac{\rho_l g R}{2} (1 - \epsilon_B - p') \right]$$

for the buffer region $\delta \leq \xi \leq \lambda$, respectively. It is noted that $\tau_c(\xi)$ linearly increases with ξ, while $\tau_B(\xi)$ monotonically decreases with ξ. Therefore, a maximum (the maximum Reynolds shear stress) is expected at $\xi = \delta$, where it is just the inversion point. In this way one might obtain an alternative expression for the inversion point. Therefore, we do not think that the velocity distribution is the only way to specify the point.

Literature cited

Burns, L. F., and R. G. Rice, "Circulation in Bubble Columns," *AIChE J.*, **43**, 1390 (1997).

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